

ηN S-wave scattering length in a three coupled channel, multiresonance, unitary model

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The S-wave scattering length for ηN elastic scattering is extracted from the S-wave T-matrix in a three coupled channel, multiresonance unitary model. Results are compared with values already reported in literature which are obtained applying multichannel, but single resonance – no background models. A dispersion among the previously published values of the real part of the S-wave scattering length is observed. We demonstrate that the reported spread originates from the strong sensitivity of the scattering length upon the small variation of the used input resonance parameters. In addition, we show that ηN scattering length value obtained in single resonance – no background models significantly increases if background term is added in a unitary way. We question the reliability of previously reported values based only on the single resonance – no background models, and demonstrate that the value of the ηN S-wave scattering length obtained in this publication is much more realistic because of the multiresonance and unitary approach.

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In 1985 Bhalerao and Liu [1] have constructed a coupled channel isobar model for the $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$ and $\eta N \rightarrow \eta N$ T - matrices with πN , ηN and $\pi\Delta$ ($\pi\pi N$) as isobars. A single resonance separable interaction model for S_{11} , P_{11} , P_{33} and D_{13} partial waves has been used. They have used only πN elastic scattering data as a constraint while their prediction for the η production cross section has been compared with, at that time the most recent data [2]. Their conclusion has been that the S-wave ηN interaction is attractive, and they have extracted for the S-wave scattering length the value of $a_{\eta N} = (0.27 + i 0.22)$ fm.

Arima et al [3] have studied the nature of the S-wave resonances $S_{11}(1535)$ and $S_{11}(1650)$ concerning their couplings with the ηN channel using the two quark-model wave functions with pure intrinsic spin states for the isobars. The dynamical coupling of the isobars to πN and ηN channels are described by the meson-quark coupling. In addition to analyzing the agreement of the model with the πN elastic and η production data they have obtained the S-wave scattering length $a_{\eta N} = (0.98 + i 0.37)$ fm.

Wilkin [4] based his calculation on an S-wave threshold enhancement calculation, used the η total cross section near threshold to fix the imaginary part of the T-matrix and obtained the real part by fitting the $\pi^- p \rightarrow \eta n$ production cross section up to the center of mass momentum in the ηn system of 1.2 (fm $^{-1}$). He quotes the value of $a_{\eta N} = (0.55 \pm 0.20 + i 0.30)$ fm.

Abaev and Nefkens [5] have also used a form of a S-wave single resonance model, adjusted the resonance parameters to reproduce the $\pi^- p \rightarrow \eta n$ production channel to the best of their ability and extracted the S-wave scattering length as: $a_{\eta N} = (0.62 + i 0.30)$ fm.

The large spread in these values of the fundamental $a_{\eta N}$ parameter requires the need for better understanding of the ηN system at low energies. The first step to achieve this is to obtain a reliable set of $\pi N \rightarrow \eta N$ T-matrices.

The indication of strong and attractive ηN interaction has led to a speculation about the existence of a new type of nuclear matter – quasi-bound η -mesic nuclei [6]. The properties of this new matter are determined by the ηN interaction at low energies.

Good data on η production in $\pi^- p$ interaction are missing. The dominant contribution to the surprisingly big η production channel is coming from the $S_{11}(1535)$

resonance. The contributions of the $P_{11}(1440)$ and $D_{13}(1520)$ resonances are important, but not completely clarified. The role of other resonances, even in these partial waves, is not at all discussed because of the single resonance character of the models [1,3,4]. Recently, accurate η photoproduction data have been obtained by TAPS at MAMI [7] up to $E_\gamma = 790$ MeV. These data indicate that the D_{13} resonance contribution is small.

Several attempts to extract the ηN S-wave scattering length $a_{\eta N}$ on the basis of complete knowledge of πN elastic channel and a part of the η production data have been made [1,3–5,8] using models of different complexity. However, with the exception of Arima et al [3] only a single resonance per partial wave has been used, therefore, producing little information about the importance of other resonances and background terms. The imaginary parts in all analyses have shown a fair amount of agreement, but the outcome for the real parts differs drastically from case to case, showing the spread from 0.27 fm [1] to 0.98 fm [3].

We shall show that the complete (and not only partial) knowledge of the ηN S-wave T-matrix is essential in order to extract the real part of the S-wave scattering length. Therefore, attempts not founded on a multichannel, multiresonance, unitary representation of the S-wave T-matrix should be considered as a rough estimate only. The S-wave scattering length, coming out of our three coupled channel, multiresonance analyses [9] gives higher values for the scattering length magnitude than it has been previously reported in all publications with the exception of Arima et. al. [3]. All values of the extracted $a_{\eta N}$ are given in Table 1.

Formalism

The formalism used in this article is a standard textbook formalism for extracting the scattering length from the known partial wave T-matrix. We give here the essential formulae for the convenience of the reader. The partial wave S matrix is written in the following way:

$$S_l = \frac{1 + i k_l(p)}{1 - i k_l(p)} ; \quad k_l(p) = \tan \delta_l(p), \quad (1)$$

p is center of mass momentum, l is the angular momentum quantum number and δ

is the partial wave phase shift.

We use the low energy expansion*:

$$p^{2l+1} \cot \delta_l = \frac{1}{a_l} + \frac{r_l}{2} p^2 + O(p^4) \quad (2)$$

where a_l and r_l are scattering length and effective range, respectively.

Using the definition of the partial wave S-matrix for the S-wave we obtain the expression for the scattering amplitude $f_0(p)$:

$$f_0(p) = \frac{T_0}{p} = \frac{1}{2ip} [S_0(p) - 1] \approx \frac{a_0}{1 - ia_0 p + \frac{1}{2}a_0 r_0 p^2} \quad (3)$$

So the final expression for the S-wave scattering length a_0 used through this article is given as:

$$a_0 = \lim_{p \rightarrow 0} \frac{f_0(p)}{1 + i p f_0(p) - \frac{1}{2} f_0(p) r_0 p^2} \quad (4)$$

and can be approximated by

$$a_0 \approx \frac{f_0(p)}{1 + i p f_0(p)} \quad (5)$$

for the small p . The $\lim_{p \rightarrow 0}$ in Eq.(4) is done numerically.

When only the elastic channel is opened, the scattering length is a real quantity. Upon opening other, inelastic channels, the scattering length becomes complex and its imaginary part is related to the total cross sections via the optical theorem. For the multichannel case [10,11] we have

$$\text{Im } f_{\alpha \rightarrow \alpha}(p_\alpha, \vartheta = 0) = \frac{p_\alpha}{4\pi} \sum_x \sigma_{\alpha \rightarrow x}^{\text{tot}} \quad (6)$$

where x denotes all opened channels. In our model we have decided to take, in addition to two physical two body channels πN and ηN , an effective two body channel $\pi^2 N$ which collects all remaining two and many body contributions. In case of s wave scattering at ηN threshold ($f_0 \approx a_0$), we get

*Different textbooks take opposite sign for the scattering length, see [10] vs [11]. Here we follow Ref. [10].

$$Im\ a_0(\eta N) = \frac{1}{4\pi} \lim_{p_\eta \rightarrow 0} p_\eta \left(\sigma_{\eta N \rightarrow \pi N}^{tot} + \sigma_{\eta N \rightarrow \pi^2 N}^{tot} \right) \quad (7)$$

where p_η is the c.m. momentum of the particles in the ηN channel. Note that the contribution of ηN elastic scattering is zero because $\lim_{p_\eta \rightarrow 0} p_\eta \sigma_{\eta N \rightarrow \eta N}^{tot} = 0$. For the other two channels the p_η factor appears in the denominator of the expression for the total cross section because of the input flux factor, and that leads to the finite contribution to the imaginary part of the scattering length.

Using isospin algebra and detailed balance we get the lower bound for the imaginary part of the S-wave scattering length:

$$Im\ a_0(\eta N) \geq \frac{3p_\pi^2}{8\pi} \frac{\sigma_{\pi^- p \rightarrow \eta n}^{tot}}{p_\eta}, \quad (8)$$

where the p_π is the c.m. momentum of the particles in the πN channel at ηN threshold. Using the experimental value

$$\frac{\sigma_{\pi^- p \rightarrow \eta n}^{tot}}{p_\eta} = (21.2 \pm 1.8) \text{ } \mu\text{b/MeV} \quad (9)$$

from Ref. [12] we obtain the optical theorem constraint based on the experimental $\sigma_{tot}(\pi N \rightarrow \eta N)$ value:

$$Im\ a_0(\eta N) \geq (0.24 \pm 0.02) \text{ fm.} \quad (10)$$

Keeping in mind that for the lowest S_{11} resonance the branching ratio to $\pi^2 N$ channel (πN , $\pi\Delta$ and others) is 5–20 % [13], we make an estimate that $Im\ a_0(\eta N) \approx 0.30$ fm. However, let us mention that our analysis [9] prefers somewhat smaller branching ratio, we get 3 ± 3 % which agrees with the value quoted in [14]. The identical method of using the unitarity to extract the lower bound for the imaginary part of the ηN scattering length has been used in Ref. [4].

Previously reported S-wave scattering length values

The ηN S-wave scattering length has been extracted from πN elastic data using different forms of multichannel single resonance models with number of channels reduced to two. The constraint to the $\pi N \rightarrow \eta N$ total cross section is imposed

in [4], while other authors have only compared the outcome of their analysis with η production total cross section [3–5]. Only authors in Ref. [1] compare their result with the η production differential cross section measurements. We collect all reported values [1,3–5,8] in Table 1.

All extracted values for the imaginary part agree reasonably well in spite of the differences of the used models (previously described). This is a direct consequence of the fact that the optical theorem via unitarity is build into each of them. Let us point out that only one analysis [1] violates the experimental optical theorem value given in Eq. (10), and we will now explain why, and how it can be modified. In the model of Ref. [1] the πN elastic scattering has been used to constrain all free channel parameters of the analysis, including ηN channel. Therefore, the $\pi N \rightarrow \eta N$ differential cross section is a prediction. The obtained predicted values have been compared only to the experimental data of Ref. [2] at energies up to 1572 MeV. These data tend to be much lower than the results of other measurements. Since data of Ref. [2] are now considered to be questionable, there is a need to correct the obtained scattering length value in order to improve the agreement with the recently recommended [15] $\pi^- p \rightarrow \eta n$ data.

A simple estimate of how much a scattering length, quoted in [1], will change if the condition to reproduce the recommended [15] $\pi^- p \rightarrow \eta n$ cross sections is imposed, can be done in the following way:

If a restriction of using only one resonance per partial wave is imposed (and [1] is a single resonance model), the ηN total production cross section very close to the threshold is given by:

$$\frac{\sigma_{\pi^- p \rightarrow \eta n}^{tot}}{p_\eta} \approx \frac{2}{3} \frac{4\pi}{p_\pi^2} |T_{\pi\pi}| |a_{\eta N}|. \quad (11)$$

This gives the values of $15.0 \mu b/\text{MeV}$ and $13.4 \mu b/\text{MeV}$ for each of the two solutions of Ref. [1] respectively. If a value for the πN elastic S-wave T-matrix at threshold are read of the graph from Ref. [1], and are taken to be : $T_{\pi\pi}^1 = (0.38 + i 0.31)$ and $T_{\pi\pi}^2 = (0.37 + i 0.25)$ the correction factors M_j ; $j=1,2$ needed to obtain the recommended cross section values given in Eq. (9) turn out to be:

$$M_j = \begin{cases} 1.41 & \text{for the solution } (0.27 + i 0.22); \quad j=1 \\ 1.58 & \text{for the solution } (0.28 + i 0.19); \quad j=2 \end{cases} \quad (12)$$

The resulting "modified" ηN scattering length solutions are:

$$\begin{aligned} (0.27 + i 0.22) \text{ fm} &\longrightarrow (0.38 + i 0.31) \text{ fm} \\ (0.28 + i 0.19) \text{ fm} &\longrightarrow (0.44 + i 0.30) \text{ fm} \end{aligned} \quad (13)$$

Therefore, if a consistent fit of $\pi^- p \rightarrow \eta n$ data is used, the outcome for imaginary part of the ηN scattering length in Ref [1] becomes consistent with the value given in Eq. (10).

Real part of the $a_{\eta N}$, however, shows a notable spread among the models ($0.27 \leq \text{Real}(a_{\eta N}) \leq 0.98$) in spite of the fact that almost identical data have been used as input. The origin of this disagreement has not been identified up to now. The aim of this paper is to address and to explain this disagreement within the framework of single resonance models, and to give the value for the $a_{\eta N}$ when the extension to multiresonance unitary models with background explicitly included is done.

Single S-wave resonance model (SR)

The single resonance model (SR) can be introduced in at least two ways:

- a.** to use the resonance parameters [13,14] directly
- b.** to simulate a single resonance model within the scope of three coupled channel, multiresonance, unitary model [9]

We have tested both approaches. As it is to be expected, the outcome is very similar.

We have tested the behavior of the ηN scattering length with respect to the probable uncertainty of resonance parameters in a single resonance model. We have used the resonance parameters [13,14], and we have allowed for their variation in the following way:

$$\begin{aligned} M_R &= (1535 \pm 10) \text{ MeV} \\ \Gamma &= (150 \pm 20) \text{ MeV} \\ x_\pi &= (0.4 \pm 0.05) \end{aligned} \quad (14)$$

Instead of using the ηN branching ratio x_η explicitly, we have used the fact that it is proportional to $\sigma_{tot}(\pi N \rightarrow \eta N)$ in all single resonance models, and we have taken a directly measured value of total cross section as the input parameter. As an illustration, the value given in Eq. (9) for the $\pi^- p \rightarrow \eta n$ total cross section together with values from Eq. (14) gives

$$x_\eta = 0.405 \pm 0.023. \quad (15)$$

Note that in [13] only a band of values* instead of real statistical error is given for all resonance parameters. The chosen variations in Eqn. (14) are, therefore, an expression of our decision, and are intended to show the sensitivity of scattering length to a relatively small changes of input values. Of course, our choice is within the suggested bands.

Using the Cutkosky's unitary formalism [18] we have obtained the S-wave T-matrix in the single resonance model using the parameter values from (14). The corresponding S-wave scattering length is obtained as the $p = 0$ value of the scattering amplitude $f_0(p)$ defined in Eq. (3). The numerical value of the scattering length with the uncertainties coming from the allowed variations of the resonance parameters is given as follows:

$a_{\eta N}$	$(0.404 + i 0.343)$ fm			
$\pm (0.085 + i 0.046)$ fm	using	± 10 MeV		for M
$\pm (0.053 + i 0.020)$ fm	using	± 20 MeV		for Γ
$\pm (0.050 + i 0.023)$ fm	using	± 0.05		for x_π
$\pm (0.034 + i 0.018)$ fm	using	$\pm 1.8 \mu b/MeV$	for $\frac{\sigma_{tot}(\pi^- p \rightarrow \eta n)}{p_\eta}$	
$\pm (0.117 + i 0.058)$ fm	Total			

(see Table 1.).

The dependence of ηN scattering length on M , Γ , x_π and $\frac{\sigma_{tot}(\pi^- p \rightarrow \eta n)}{p_\eta}$ is shown on Figures 1a–1d. One parameter is variable, and other three are fixed to the values

* Citation from [13] follows: Resonance mass $M = 1520$ to 1555 (≈ 1535) MeV, full width $\Gamma = 100$ to 250 (≈ 150) MeV, fraction Γ_i / Γ is 35–50 % for πN , 30–50 % for ηN and 5–20 % for $\pi\pi N$ channel. Let us still point out that in Ref [14] the suggested value for the π^2 branching ratio is only 1 – 10 %.

given in Eqs. (9) and (14). Error bands on Figures 1a–1d were obtained using the standard statistical definition of the total error of the function which depends on several uncertain parameters. Upon closer inspection of Figures 1a–1d we conclude that the small variation of input parameters causes a big change in the resulting scattering length. We conclude that the present level of confidence of S_{11} resonance parameters and constraint on $\pi N \rightarrow \eta N$ cross section, see Eq. 9, is not sufficient to predict the ηN scattering length to a level better than 50 %.

Fig 2a shows the comparison of the obtained single resonance πN elastic T-matrix, corresponding to the parameter values from Eqs. (9) and (14), with the partial wave analyses of Höhler et. al. [19]. As the K-H PWA does not give the error analysis for the partial wave T-matrices in [19], and the errors are essential to define the statistical weight of the analyses, we have identified the errors of the used data in the standard χ^2 analysis as:

$$\Delta_i = 0.005 + \left(0.01 + 0.0015 \frac{W_i - W^{\pi \text{ thresh}}}{\Delta} \right) |T_{\max}|$$

$$\Delta = 1 \text{ GeV}$$

W_i is the total c.m. energy

$W^{\pi \text{ thresh}}$ is the total energy at π nucleon threshold

$|T_{\max}|$ is the maximal value of the S_{11} T – matrix in the chosen energy range.

The energy range extends up to 2.5 GeV.

The statistical weight in the χ^2 function is defined in a standard way:

$$w_i = \frac{1}{(\Delta_i)^2}.$$

The introduced energy dependence of the statistical weight is inspired by the energy dependence of the error analysis of Ref. [18]. It steadily raises with energy, but does not exceed the value of 0.02 in the units of Ref. [19].

The scattering amplitude in the SR model is given in Fig. 3 and denoted with the dotted line, the extrapolation to the $p_\eta = 0$ value (the S-wave scattering length) is marked with the empty triangle on the y-axes.

The agreement between predictions for the πN elastic T-matrices, within the scope of *any* single resonance model, and the standard multiresonance πN elastic input of Ref. [19] is acceptable only in the vicinity of $S_{11}(1535)$ dominance. That

indicates problems occurring in any S-wave single resonance model in reproducing the "experimental" πN elastic scattering length* because of the disagreement of the obtained T-matrices with well known input near π threshold. To demonstrate the problem we compare the "experimental" [19] and SR model values for the πN elastic S-wave scattering lengths:

$$a_{\pi N}^{exp} = (0.249 \pm 0.004) \text{ fm} \quad \text{and} \quad a_{\pi N}^{SR \ mod} = 0.066 \text{ fm}.$$

The discrepancy is obvious.

Conclusion: *The result of our SR model is, as it is to be expected, consistent with the results of all other single resonance models [1,4,5]. A single resonance model which is based on Particle data group data for the lowest S-wave resonance [13], and constrained with η production total cross section [12] can not be reliably used for extracting the ηN S-wave scattering length. It is extremely sensitive to the precision of resonance input parameters, and has notorious problems in failing to reproduce the πN elastic scattering length. The simplest improvement consists in including the background term in addition to a single resonance.*

Single S-wave resonance + one background term model (SRBG)

To improve the model and to manifestly demonstrate the importance of additional ingredients we have introduced one background term in addition to the SR model. The πN elastic S-wave scattering length, and high energy behavior of πN elastic S-wave T-matrix can not be even remotely described without introducing the background into the πN elastic channel. Of course, in a unitary model any modification in one channel will influence other channels as well. So, consequently, the addition of the background term to the ηN channel is needed when the background

* As it is pointed out in Ref. [19] at least some form of model dependence have to be introduced in extrapolating the scattering amplitude to the πN elastic threshold. So, there can exist no such quantity as "experimental" scattering length.

term is added to the πN elastic channel. Therefore, changes in the S-wave scattering lengths in both channels are to be expected.

We have used the formalism described in [9]. However, as we are interested in the S-wave only, we perform here a fit of the background term parameters to the S_{11} πN elastic T -matrix [19] up to the total c.m. energy of 1560 MeV where the importance of $S_{11}(1535)$ resonance diminishes. A constraint to the ηN channel is not yet imposed. Note that because of the formalism used, releasing the background parameters in the fitting procedure also slightly changes resonance parameters. The obtained parameters are:

$$\begin{aligned} M^{res} &= 1538 \text{ MeV} \\ \Gamma^{res} &= 127 \text{ MeV} \\ x_\pi^{res} &= 0.33 \\ x_\eta^{res} &= 0.49 \\ \frac{\sigma_{tot}(\pi N \rightarrow \eta N)}{p_\eta} &= 9.2 \mu\text{b}/\text{MeV} \end{aligned} \tag{16}$$

Resulting T-matrix (full curves) is compared to Höhler PWA (full dots) [19] in Fig. 2b. Dashed and dotted curves represent the resonance and background contributions, respectively.

The corresponding S-wave scattering length is obtained as the $p = 0$ value of the scattering amplitude $f_0(p)$ defined in Eq. (3). The scattering amplitude in the SRBG model is given in Fig. 3 and denoted with the dashed line, the extrapolation to the $p_\eta = 0$ value (the S-wave scattering length) is marked with the empty inverse triangle on the y-axes. The numerical value of the scattering length resulting from the resonance parameters of Eq. (16) is given by:

$$a_{\eta N} = (0.691 + i 0.174) \text{ fm}$$

(see Table 1.). As the SRBG model is a simplification of our full, multiresonance model [9], and is given only as a demonstration of importance of different parts of the model. For simplicity, the error analysis is not given for SRBG. However it is identical to the error analysis of our full model presented in Ref. [9], and will be given in extracting the ηN S-wave scattering length, in the final step, within the scope of our final CCMRU model.

Real part of the scattering length shows the strong tendency of rising (0.398 in SR vs 0.685 in SRBG). It is interesting to point out that fitting only the elastic πN part *strongly reduces the total cross section of η production : 21.2 in SR vs 9.2 in SRBG*, producing the analogous reduction of the scattering length imaginary part $Im\ a_{\eta N}$: $0.343\ fm$ in SR vs $0.174\ fm$ in SRBG.

The agreement of the SRBG model with the standard multiresonance πN elastic T-matrix of Ref. [19] is now much better near the πN threshold. Therefore, the agreement of the πN elastic scattering length within the SRBG model with the "experimental" value is much better. To illustrate the effect we compare the "experimental" and SRBG model values for the πN elastic S-wave scattering lengths:

$$a_{\pi N}^{exp} = (0.249 \pm 0.004) \text{ fm} \quad \text{and} \quad a_{\pi N}^{SRBG\ mod} = 0.259 \text{ fm}.$$

However, the result for $\sigma_{tot}(\pi^- p \rightarrow \eta n)$, obtained in the SRBG model, is not acceptable. It completely disagrees with the experimental results. Therefore, the unavoidable next step is releasing the resonance and background parameters in the fitting procedure imposing a constraint on the η production cross section at the same time.

Conclusion: *Introducing background term modifies the ηN scattering length significantly and the πN elastic scattering length is improved. However, the free fit to πN data without any constraint to the ηN channel does not reproduce the η production total cross section at all. Therefore, a constraint of the fit with the ηN channel data is needed.*

Constrained single resonance + one background model (CSR BG)

Again, we have used the coupled channel, multiresonance and unitary formalism presented in Ref. [9]. The resonance and the background parameters have been simultaneously released in the fitting procedure, and we have fitted the S_{11} πN elastic S-wave T-matrix of Ref. [19] up to the total c.m. energy of 1560 MeV. The ηN channel is constrained by forcing the total $\pi N \rightarrow \eta N$ total cross section at threshold to agree with the experimental value. That has been done in a standard way introducing a penalty function into χ^2 .

Resulting πN elastic S-wave T-matrix (full curve) is compared to Höhler's PWA (full dots) in Fig. 2c and the ηN scattering length is given in Table 1. Dashed and dotted curves represent the resonance and background contributions, respectively. The obtained resonance parameters have not changed drastically except, of course, x_η :

$$\begin{aligned}
M^{res} &= 1537 \text{ MeV} \\
\Gamma^{res} &= 145 \text{ MeV} \\
x_\pi^{res} &= 0.29 \\
x_\eta^{res} &= 0.70 \\
\frac{\sigma_{tot}(\pi N \rightarrow \eta N)}{p_\eta} &= 21.2 \mu\text{b}/\text{MeV}
\end{aligned} \tag{17}$$

The corresponding S-wave scattering length is obtained identically as before, as the $p = 0$ value of the scattering amplitude $f_0(p)$ defined in Eq. (3). The scattering amplitude in the CSRBG model is given in Fig. 3 and denoted with the dash-dotted line, the extrapolation to the $p_\eta = 0$ value (the S-wave scattering length) is marked with the empty square on the y-axes. The numerical value of the scattering length resulting from the resonance parameters of Eq. (17) is given by:

$$a_{\eta N} = (0.968 + i 0.281) \text{ fm}$$

(see Table 1.). The CSRBG model is a simplification of our full, multiresonance model [9], and is given only as a demonstration of importance of different parts of the model. *

Because of better agreement between the CSRBG model and the input πN elastic S-wave T-matrices, the value for the πN elastic S-wave scattering length is improved.

$$a_{\pi N}^{exp} = (0.249 \pm 0.004) \text{ fm} \quad \text{and} \quad a_{\pi N}^{CSRBG \ mod} = 0.251 \text{ fm}.$$

Conclusion: *Introducing the background term in a unitary way, and reproducing all experimental inputs, shifts the real part of the scattering length to higher values. The πN elastic scattering length is closer to the "experimental" value. The agreement of*

*The error analysis is not given for the same reasons as for SRBG model.

the elastic πN T-matrix with data of Ref. [19] is not yet perfect, so the inclusion of other S-wave resonances and background terms is needed.

Three coupled channel, multiresonance and unitary model (CCMRU)

We have constructed a three coupled channel, multiresonance and unitary model (CCMRU) [9], and fitted it to the πN elastic partial wave T-matrices of Ref. [19] in 8 lowest $I = 1/2$ partial waves: S_{11} , P_{11} , P_{13} , D_{13} , D_{15} , F_{15} , F_{17} and G_{17} , up to total c.m. energy of 2500 MeV, and to the available $\pi^- p \rightarrow \eta n$ total and differential cross sections. The results of the analysis are presented in Ref. [9], with all resonance parameters explicitly given therein. For the convenience of the reader, the comparision of the S_{11} πN elastic T-matrix with the input data of [19] is given in Fig. 2d.

The corresponding S-wave scattering length is obtained identically as before, as the $p = 0$ value of the scattering amplitude $f_0(p)$ defined in Eq. (3). The scattering amplitude in the CCMRU model is given in Fig. 3 and denoted with the full curve, the extrapolation to the $p_\eta = 0$ value (the S-wave scattering length) is marked with the empty circle on the y-axes. The numerical value of the scattering length, with the error analysis given in Ref. [9], is given by:

$$3 \text{ resonances in } P_{11} \text{ partial wave: } [0.886 \pm 0.047 + i(0.274 \pm 0.039)] \text{ fm}$$

$$4 \text{ resonances in } P_{11} \text{ partial wave: } [0.876 \pm 0.047 + i(0.274 \pm 0.039)] \text{ fm}$$

and we recommend it as being more reliable then the values obtained by previous analyses. The obtained πN scattering length is in complete agreement with the "experimental" value:

$$a_{\pi N}^{exp} = (0.249 \pm 0.004) \text{ fm}$$

and

$$3 \text{ resonances in } P_{11} \text{ partial wave: } a_{\pi N}^{CCMRU \ mod} = (0.247 \pm 0.006) \text{ fm}$$

$$4 \text{ resonances in } P_{11} \text{ partial wave: } a_{\pi N}^{CCMRU \ mod} = (0.248 \pm 0.006) \text{ fm}$$

Conclusion:

We do not consider our result to be the final one because of its model dependence

and because of the insufficient ηN input. However, it is certainly more reliable then the values reported within the scope of previous models. The new experiments near threshold ηN data are badly needed. Proposed [20] measurements of the total an the differential cross section for the reaction $\pi^- p \rightarrow \eta n$ from threshold ($p_\pi = 685$ MeV/c) up to $p_\pi = 760$ MeV/c will result in a better experimental basis for the reliable extraction of the ηN scattering length. The accurate determination of the S-wave ηN scattering length will shed light on the possible existance of a new kind of hadronic bound state. Namely, the indication of strong and attractive ηN interaction has led to a speculation about the existence of a new type of nuclear matter, quasi-bound η -mesic nuclei [6]. The properties of this new matter are determined by the ηN interaction at low energies.

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TABLE CAPTIONS

Table 1. *Table 1.* Values of the S-wave scattering length for various models.

FIGURE CAPTIONS

Fig. 1. Dependence of the S-wave scattering length in a single resonance model on different input parameters:

- a. Dependence on resonance mass. For other parameters influencing $a_{\eta N}$ we have the following values: resonance width $\Gamma = (150 \pm 20)$ MeV, πN branching ratio $x_\pi = (0.40 \pm 0.05)$ and $\sigma_{\pi^- p \rightarrow \eta n}^{tot}/p_\eta = (21.2 \pm 1.8)$ $\mu b/\text{MeV}$.
- b. Dependence on resonance width. Values used for resonance mass $M = (1535 \pm 10)$ MeV, resonance width $\Gamma = (150 \pm 20)$ MeV, πN branching ratio $x_\pi = (0.40 \pm 0.05)$ and $\sigma_{\pi^- p \rightarrow \eta n}^{tot}/p_\eta = (21.2 \pm 1.8)$ $\mu b/\text{MeV}$.
- c. Dependence on πN branching ratio. Values used for resonance mass $M = (1535 \pm 10)$ MeV, resonance width $\Gamma = (150 \pm 20)$ MeV and $\sigma_{\pi^- p \rightarrow \eta n}^{tot}/p_\eta = (21.2 \pm 1.8)$ $\mu b/\text{MeV}$.
- d. Dependence on $\pi^- p \rightarrow \eta n$ total cross section near ηN threshold. Values used for resonance mass $M = (1535 \pm 10)$ MeV, resonance width $\Gamma = (150 \pm 20)$ MeV and πN branching ratio $x_\pi = (0.40 \pm 0.05)$.

Fig. 2.

- a. Comparison of the πN elastic S-wave T-matrix obtained in our SR model (dashed curve) with the Höhler partial wave analysis (full dots) [19]. The used PWA does not give the error analyses for the partial wave T-matrices in [19], so the error bars given in the figure are defined in the text and reflect the statistical weight of the data set used in the minimization procedure.
- b. Comparision of the πN elastic T-matrix (full curve) obtained in our SRBG model with the Höhler partial wave analysis (full dots) [19]. The dashed curve represents the resonance contribution, while the dotted curve gives the background.
- c. Comparision of the πN elastic T-matrix (full curve) obtained in our CSRBG model with the Höhler partial wave analysis [19]. The dashed curve represents

the resonance contribution, while the dotted curve gives the background.

d. Comparision of the πN elastic T-matrix obtained in the CCMRR model with the Höhler partial wave analysis [19].

Fig. 3. The dependence of the ηN scattering amplitude upon the η momentum. The dotted, dashed, dash-dotted and full lines represent prediction of SR, SRBG, CSRBG and CCMRU models, respectively. Triangle, inverse triangle, square and open circle at the y-axes show the ηN S-wave scattering length values obtained by numerical extrapolation of corresponding scattering amplitudes.

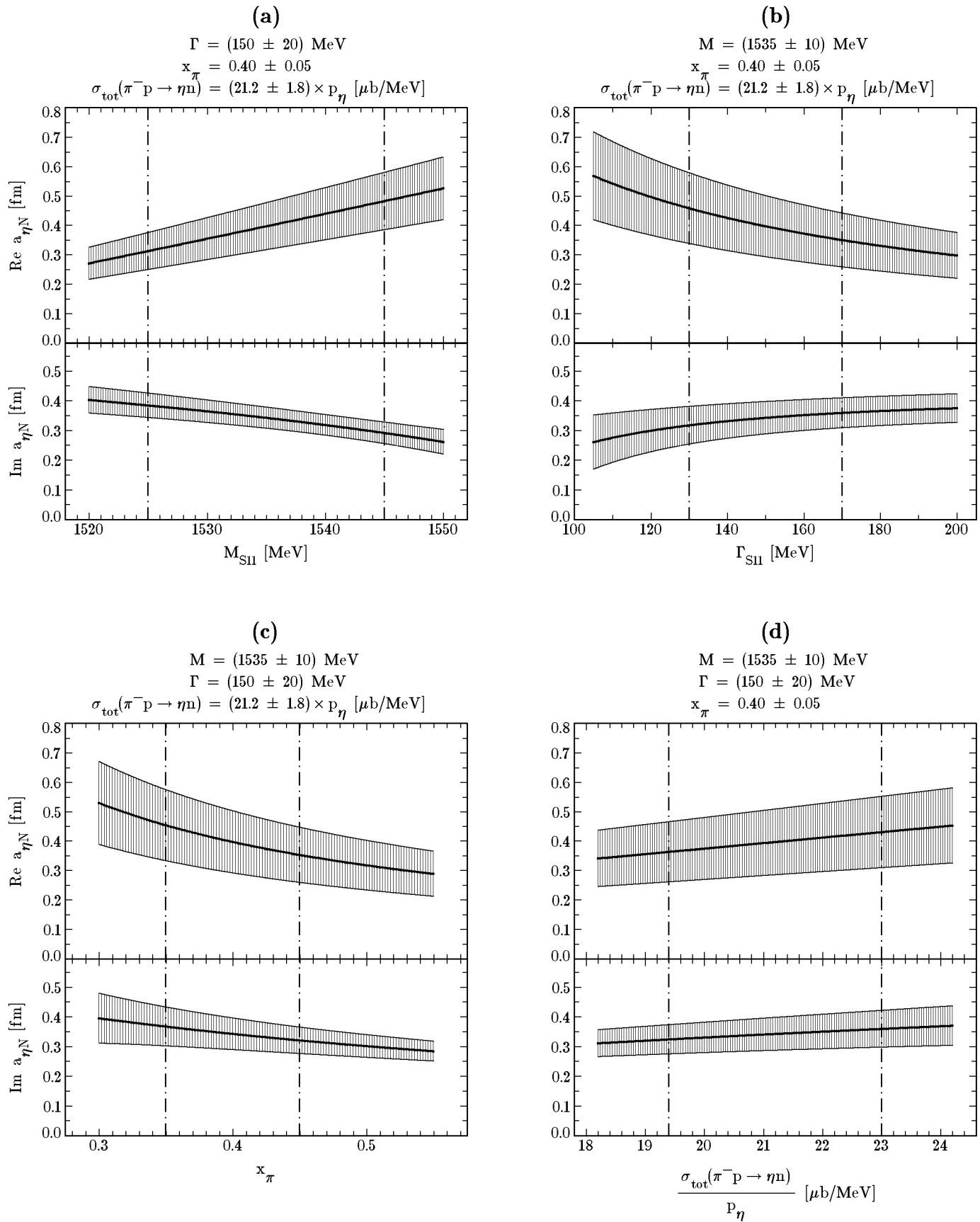
Table 1

	$Re\ a_{\eta N}$ [fm]	$Im\ a_{\eta N}$ [fm]
Bhalerao-Liu [1,8]	0.27	0.22
	0.28	0.19
"modified" Bhalerao-Liu	0.38	0.31
	0.44	0.30
Arima et al. [3]	0.98	0.37
Wilkin [4]	0.55 ± 0.20	0.30
Abaev and Nefkens [5]	0.62	0.30
SR of this paper	0.404 ± 0.117	0.343 ± 0.058
SRBG of this paper	0.691	0.174
CSRBG of this paper	0.968	0.281
CCMRU model with 3R in P_{11}	0.886 ± 0.047	0.274 ± 0.039
CCMRU model with 4R in P_{11}	0.876 ± 0.047	0.274 ± 0.039

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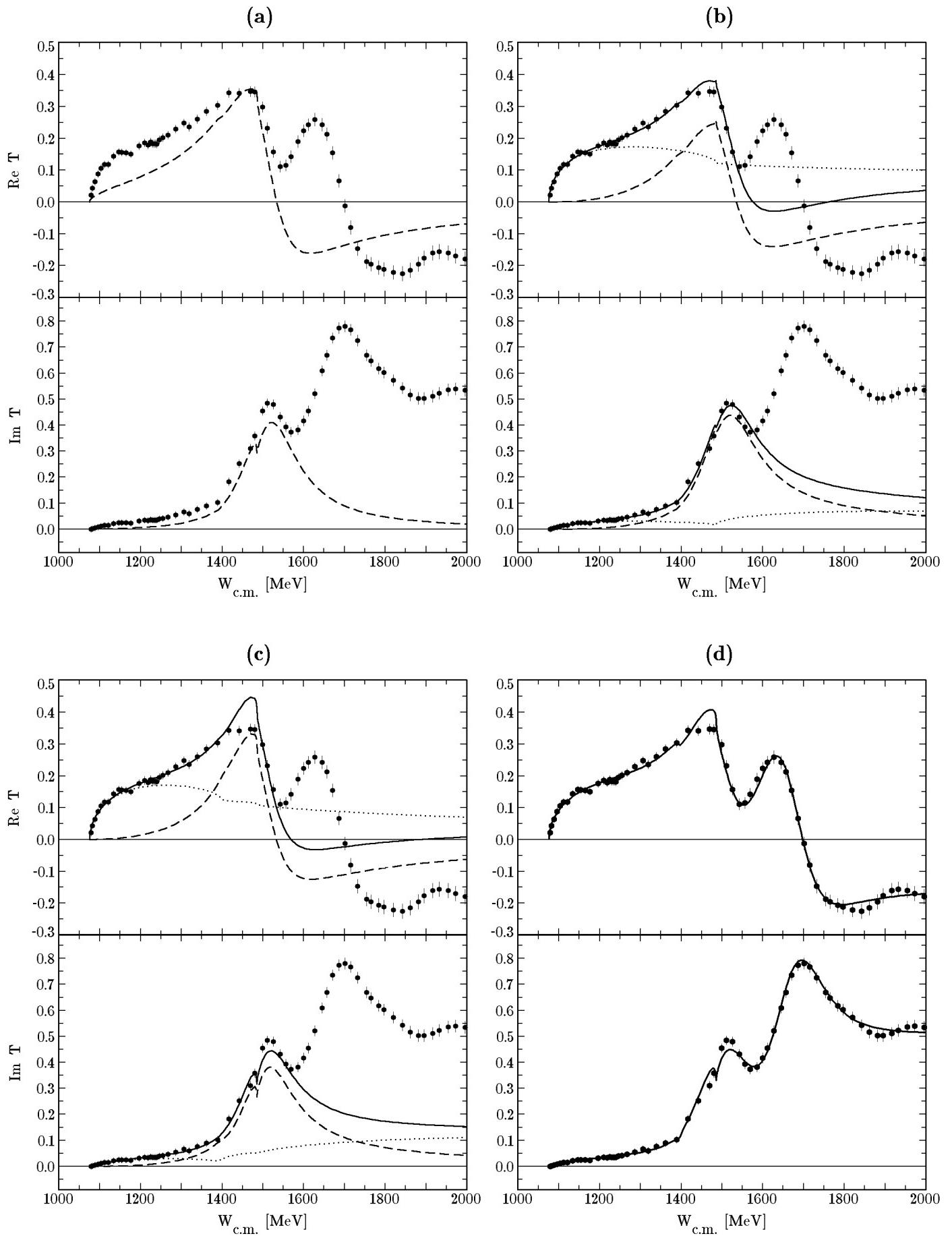
Fig. 1



This figure "fig1-2.png" is available in "png" format from:

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Fig. 2



This figure "fig1-3.png" is available in "png" format from:

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Fig. 3

